ÉCOLE NORMALE SUPÉRIEURE DE LYON

Particle Collisions in Turbulent Flows

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Laboratoire de Physique

13 December 2013

ED 52 - PHAST
Physique & Astrophysique de Lyon
École doctorale

Université de Lyon

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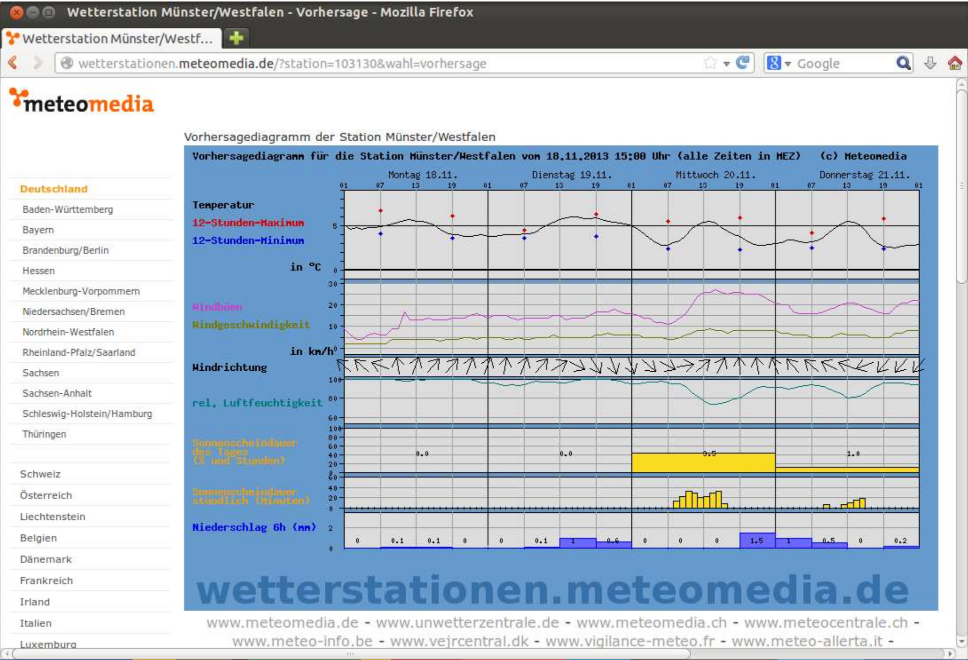

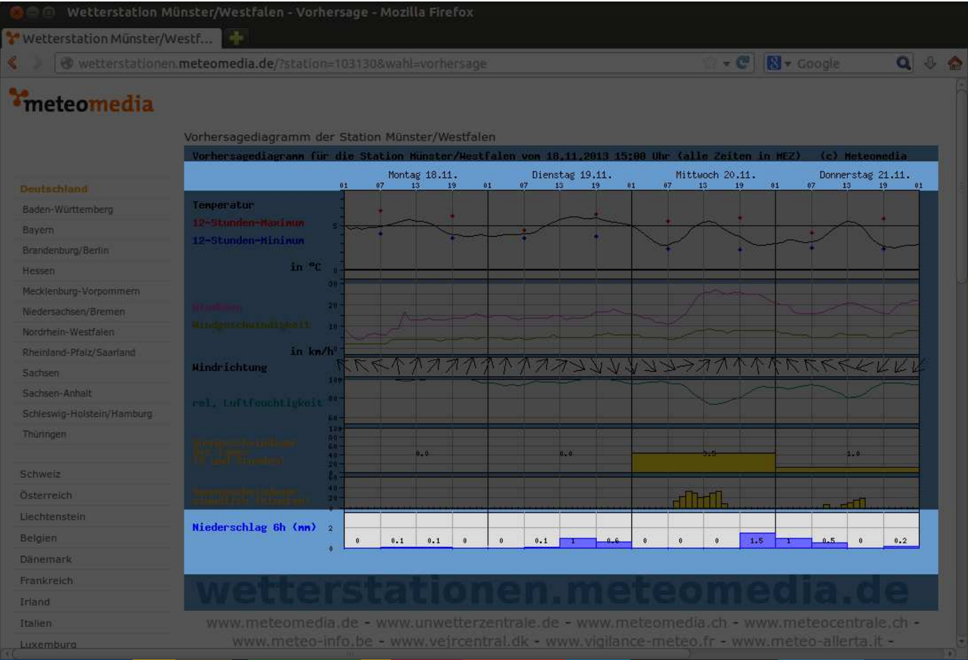
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Vorhersagediagramm der Station Münster/Westfalen
Vorhersagediagramm für die Station Münster/Westfalen vom 18.11.2013 15:00 Uhr (alle Zeiten in MEZ) (c) Meteomedia
Montag 18.11.
Dienstag 19.11.
Mittwoch 20.11.
Donnerstag 21.11.
Temperatur
12-Stunden-Maximum
12-Stunden-Minimum
in °C
in km/h
Windrichtung
Niederschlag 6h (mm)
Deutschland
Baden-Württemberg
Bayern
Brandenburg/Berlin
Hessen
Mecklenburg-Vorpommern
Niedersachsen/Bremen
Nordrhein-Westfalen
Rheinland-Pfalz/Saarland
Sachsen
Sachsen-Anhalt
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Thüringen
Schweiz
Österreich
Liechtenstein
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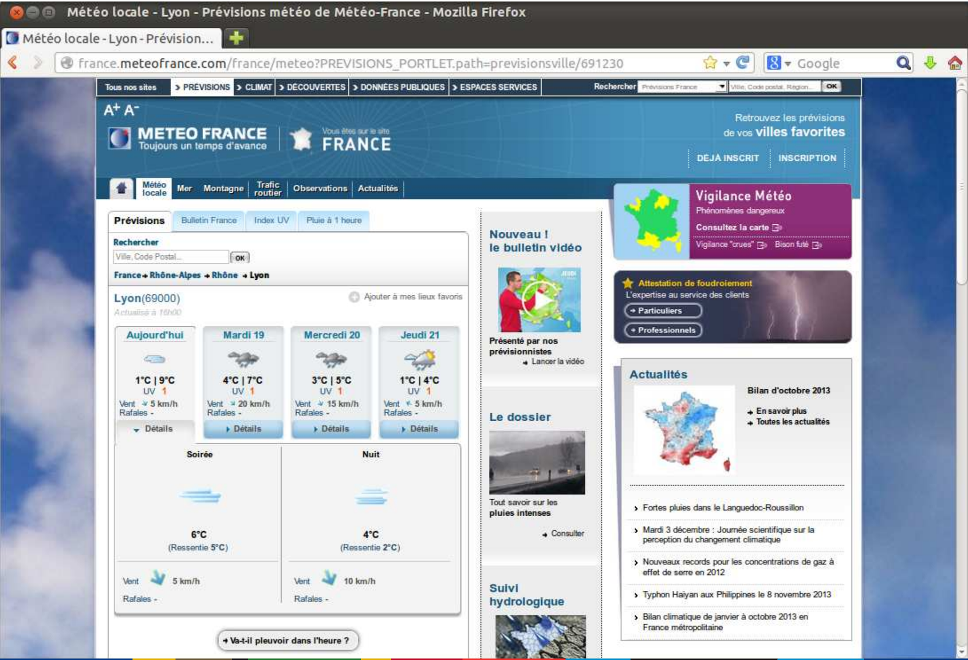
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Mer
Montagne
Trafic routier
Observations
Actualités
Vigilance Météo
Phénomènes dangereux
Consultez la carte
Vigilance "crues"
Bison futé
Prévisions
Bulletin France
Index UV
Pluie à 1 heure
Rechercher
OK
France → Rhône-Alpes → Rhône → Lyon
Lyon(69000)
Ajouter à mes lieux favoris
Aujourd'hui
Mardi 19
Mercredi 20
Jeudi 21
1°C | 9°C
4°C | 7°C
3°C | 5°C
1°C | 4°C
UV 1
Vent 5 km/h
Vent 20 km/h
Vent 15 km/h
Vent 5 km/h
Rafales -
Détails
Soirée
Nuit
6°C
(Ressentie 5°C)
4°C
(Ressentie 2°C)
Vent 5 km/h
Vent 10 km/h
Rafales -
Va-t-il pleuvoir dans l'heure ?
Nouveau ! le bulletin vidéo
Présenté par nos prévisionnistes
Lancer la vidéo
Le dossier
Tout savoir sur les pluies intenses
Consulter
Suivi hydrologique
Attestation de foudroiement
L'expertise au service des clients
Particuliers
Professionnels
Actualités
Bilan d'octobre 2013
En savoir plus
Toutes les actualités
Fortes pluies dans le Languedoc-Roussillon
Mardi 3 décembre : Journée scientifique sur la perception du changement climatique
Nouveaux records pour les concentrations de gaz à effet de serre en 2012
Typhon Haiyan aux Philippines le 8 novembre 2013
Bilan climatique de janvier à octobre 2013 en France métropolitaine

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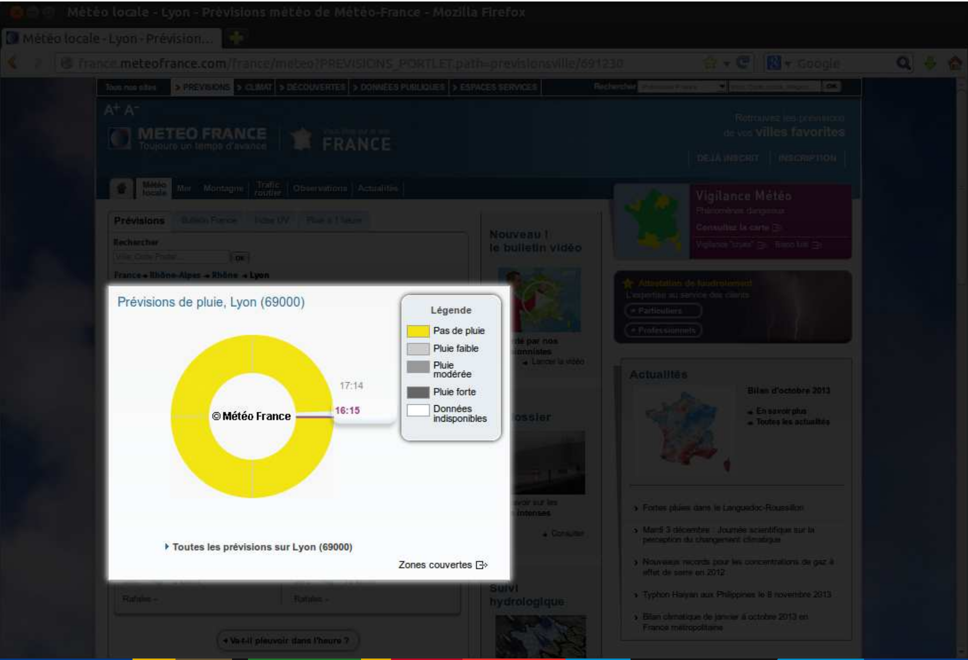
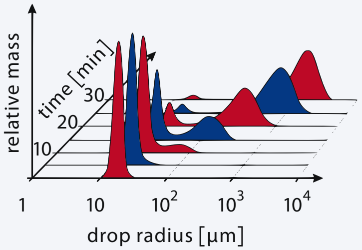


Particle Collisions in Turbulent Flows

Outline

- Introduction
- Prevalence of sling/caustics/RUM effect
- Multiple collisions
- KS vs. DNS

Rain formation and the droplet size distribution



Lamb *Meteorol. Monogr.* (2001);Shaw *ARFM* (2003)

$$\frac{\partial f(a)}{\partial t} = \text{nucleation/ condensation} + \textbf{collision/ coalescence}$$

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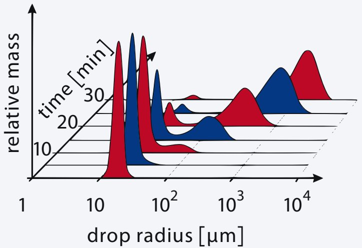
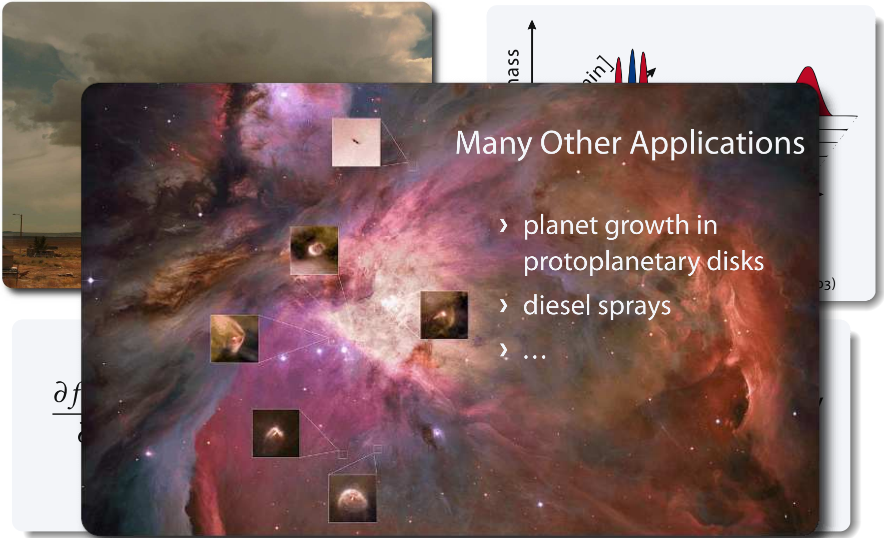


Lamb *Meteorol. Monogr.* (2001);Shaw *ARFM* (2003)

$$\frac{\partial f(a)}{\partial t} = \boxed{\text{nucleation/ condensation}} + \frac{1}{2}\int_0^a \frac{a^2}{a''^2}\Gamma(a'',a')f(a'')f(a')\,\mathrm{d}a' - \int_0^\infty \Gamma(a,a')f(a)f(a')\,\mathrm{d}a'$$

$$a''^3 = a^3 - a'^3$$

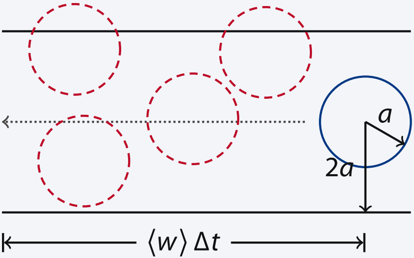
Rain formation and the droplet size distribution



$$a''^3 = a^3 - a'^3$$

Introductory example: Kinetic theory

Collision cylinder



- simplification:
  only one particle size
- collision rate for one particle

$$\mathcal{R}_c = n\,\pi(2a)^2\langle w\rangle$$

- overall collision rate

$$\mathcal{N}_c = \frac{1}{2}n^2\underbrace{\pi(2a)^2\langle w\rangle}_{\Gamma_{kin}(a)}$$

Collision kernel

$$\Gamma_{kin}(a) = \pi(2a)^2\langle w\rangle$$

(Inertial) Particle collisions in Turbulent Flows

- finite density $\rho_p > \rho_f$
- finite size $0 < a \ll \eta$
- equations of motion:

$$\frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t} = \boldsymbol{V}, \quad \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = \frac{\boldsymbol{u}(\boldsymbol{X},t) - \boldsymbol{V}}{\tau_p} + \boldsymbol{G}$$

Maxey & Riley *Phys. Fluids* (1983)
Gatignol *J. méc. théor. appl.* (1983)

- dimensionless quantity:
  Stokes number

$$\mathrm{St} = \frac{\tau_p}{\tau_K} = \frac{2}{9}\frac{\rho_p}{\rho_f}\frac{a^2}{\eta^2}$$

DNS

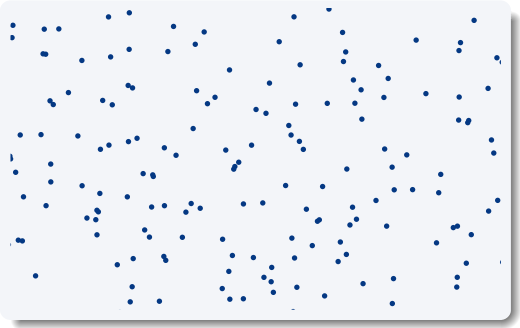
- Navier–Stokes equations
- periodic box
- 384^3 grid points
- $\mathrm{Re}_\lambda = 130$

Kinematic Simulations

- synthetic turbulence
- efficient

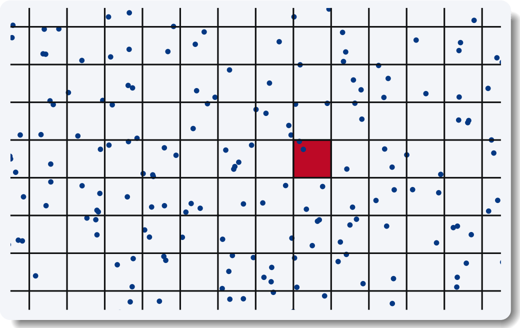
Fung et al. *JFM* (1992)

Determining the collision rate



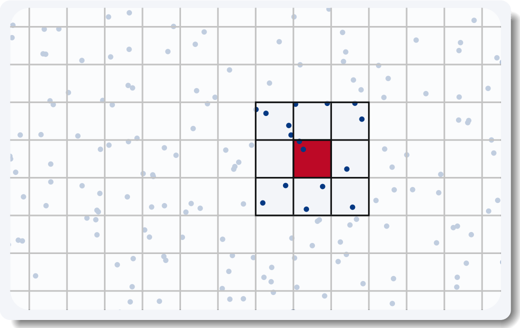
- part of simulation box with particles

Determining the collision rate



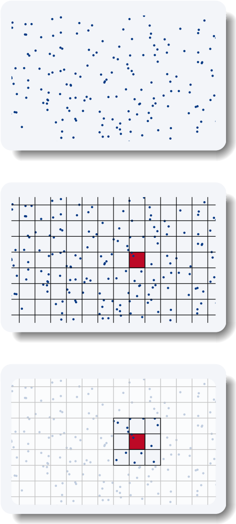
- part of simulation box with particles
- divide into segments
- know which particles in which cell

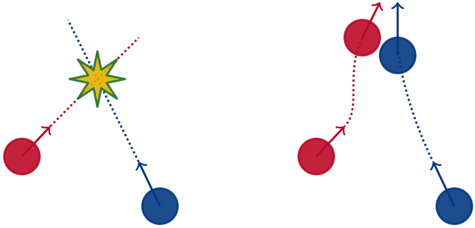
Determining the collision rate



- part of simulation box with particles
- divide into segments
- know which particles in which cell
- consider only surrounding cells

Determining the collision rate





extrapolation is inexact

rather use interpolation

Collision kernel

$$\frac{N_c(T)}{T\,V_{sys}} = \mathcal{N}_c = \frac{1}{2} n^2 \Gamma(a)$$

Prevalence of the sling/caustics/RUM effect

Saffman & Turner *JFM* (1956)

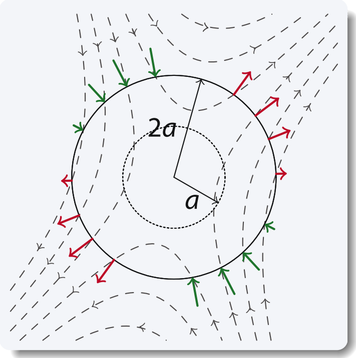
- St → 0: particles follow flow

$$\mathcal{R}_c = n \int -w_r(2a,\Omega)\Theta[-w_r(2a,\Omega)]\,d\Omega$$

- average to obtain total collision rate

$$\mathcal{N}_c = \frac{1}{2} n^2 \int \frac{1}{2}\langle |w_r(2a)| \rangle\,d\Omega$$

- approximate $\langle |w_x(2a)| \rangle = 2a\,\langle |\partial u_x/\partial x| \rangle$
- assume Gaussian statistics with $\left\langle (\partial u_x/\partial x)^2 \right\rangle = \varepsilon/15\nu$



$$\Gamma_{ST} = \left(\frac{8\pi}{15}\right)^{1/2} \frac{(2a)^3}{\tau_K}$$

Collision kernel(s)

St → 0 Saffman & Turner *JFM* (1956)

$$\Gamma_{ST} = \left(\frac{8\pi}{15}\right)^{1/2} \frac{(2a)^3}{\tau_K}$$

St → ∞ Abrahamson *Chem. Eng. Sci.* (1975)

$\Gamma_A = \Gamma_{kin}$ with $V_{\mathrm{rms}} = (\eta/\tau_K) f(\mathrm{St}, \mathrm{Re}_\lambda)$

$$\Gamma_A = 4\sqrt{\pi}(2a)^2 \frac{\eta}{\tau_K} f(\mathrm{St}, \mathrm{Re}_\lambda)$$

Collision kernel(s)

St → 0 Saffman & Turner *JFM* (1956)

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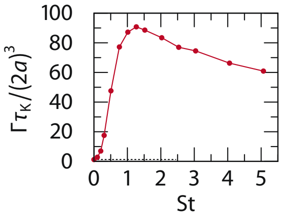
Preferential concentration ?



Sling/caustics/RUM effect ?

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$$\Gamma_A = 4\sqrt{\pi}(2a)^2 \frac{\eta}{\tau_K} f(\mathrm{St}, \mathrm{Re}_\lambda)$$

Preferential concentration

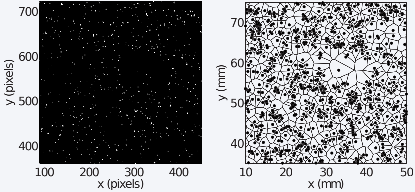
- mathematically exact

$$\Gamma_{SC} = 2\pi(2a)^2 g(2a)\langle|w_r|\rangle$$

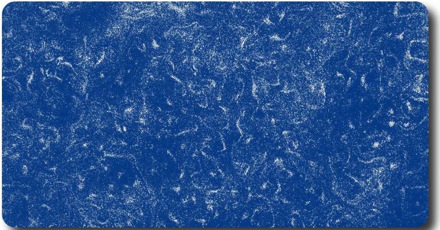
- radial distribution function $g(r)$

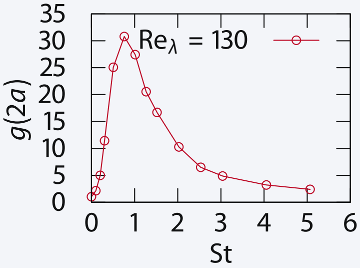
$$g(r) \sim (r/\eta)^{(\mathcal{D}_2-3)}, \quad r/\eta \ll 1$$

Sundaram & Collins *JFM* (1997)



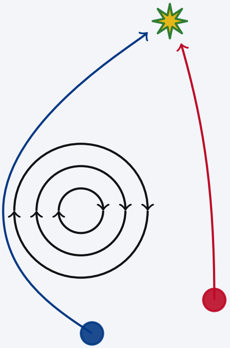
Monchaux et al. *Phys. Fluids* (2010)





Sling/caustics/RUM effect

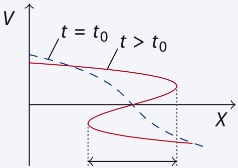
Sling



› particles slung by vortices

Falkovich et al. *Nature* (2002)

Caustics



› faster particles “overtake” slower ones

› points in phase space multi-valued

Wilkinson, Mehlig, & Bezuglyy *PRL* (2006)

RUM

Random
Uncorrelated
Motion

› two contributions to motion of inertial particles
  » smooth spatially correlated movement
  » random uncorrelated movement

Simonin et al. *Phys. Fluids* (2006)
Reeks et al. *Proc. FEDSM* (2006)

$$\Gamma = \underbrace{\Gamma_{ST}}_{\sim a^3}\ \underbrace{g(2a)}_{\sim a^{\mathcal{D}_2-3}} + \underbrace{\Gamma_A h_S(\mathrm{St}, \mathrm{Re}_\lambda)}_{\sim a^2}$$

$$\mathrm{St} = \frac{2}{9}\frac{\rho_p}{\rho_f}\frac{a^2}{\eta^2}$$

△	$\rho_p/\rho_f = 250$	$\Rightarrow$	$a = 2a_0$
□	$\rho_p/\rho_f = 1000$	$\Rightarrow$	$a = a_0$
○	$\rho_p/\rho_f = 4000$	$\Rightarrow$	$a = \frac{1}{2}a_0$

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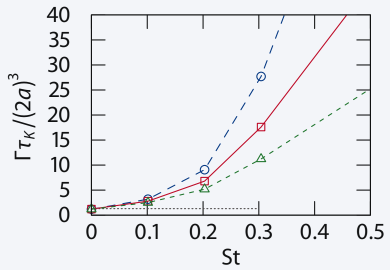
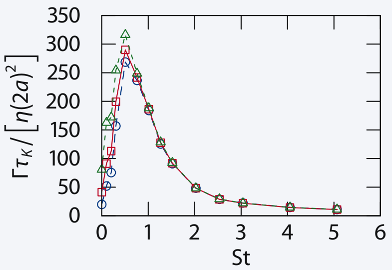
Saffman & Turner



Sling/caustics/RUM



Voßkuhle et al. arXiv: 1307.6853 [physics.flu-dyn]

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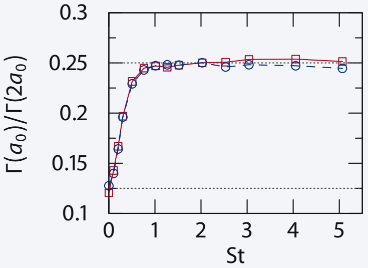
- Expected values for $\Gamma(a_0)/\Gamma(2a_0) = \Gamma(a_0/2)/\Gamma(a_0)$

$\sim a^2 \quad \Rightarrow \quad \Gamma(a_0)/\Gamma(2a_0) = \frac{1}{4}$

$\sim a^3 \quad \Rightarrow \quad \Gamma(a_0)/\Gamma(2a_0) = \frac{1}{8}$



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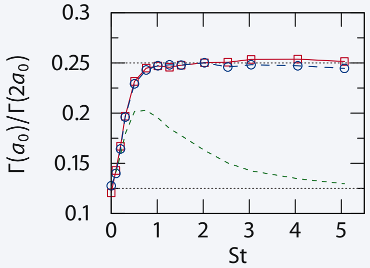
$$\sim a^3 a^{\mathcal{D}_2-3} \quad \Rightarrow \quad \Gamma(a_0)/\Gamma(2a_0) = \left(\tfrac{1}{2}\right)^{\mathcal{D}_2}$$



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Collision velocities

- Cumulative PDF of relative velocity

$$F(|w_r|) = \int_0^{|w_r|} p(|w_r|)\mathrm{d}w_r$$

"How many particle pairs have relative velocities smaller than $|w_r|$?"

- Cumulative distribution of the flux

$$\varphi(|w_r|) = \int_0^{|w_r|} \frac{|w_r|}{\langle |w_r| \rangle} p(|w_r|)\mathrm{d}w_r$$

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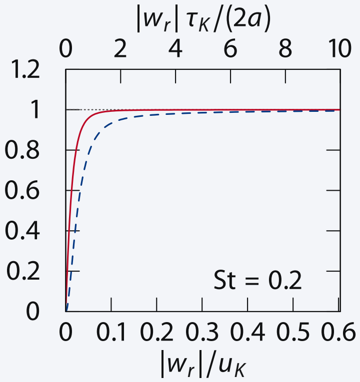

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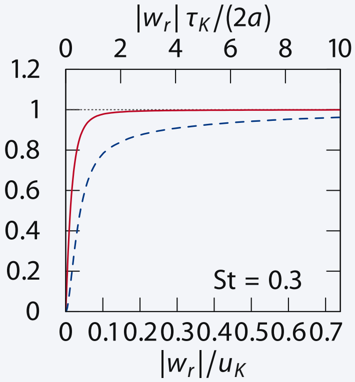

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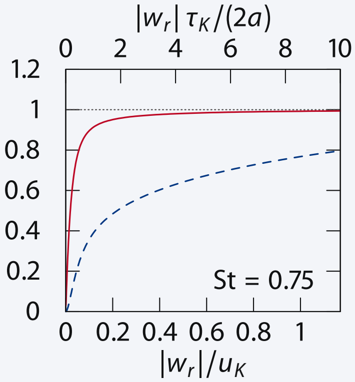

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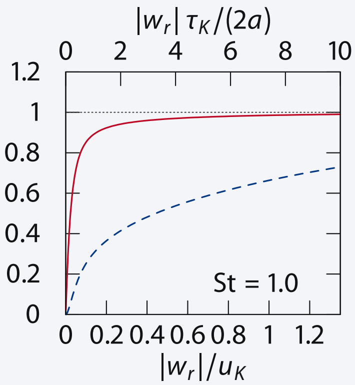

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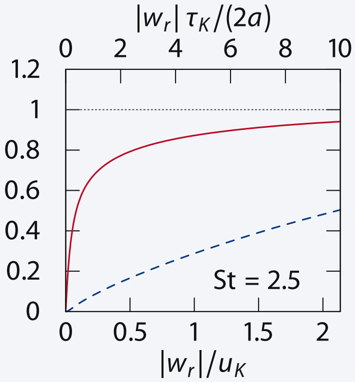

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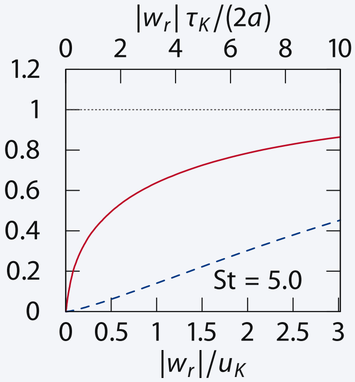
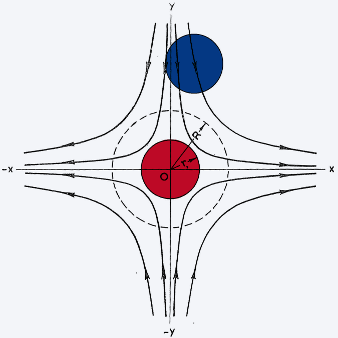


Multiple collisions

Ghost collisions and the Saffman–Turner theory



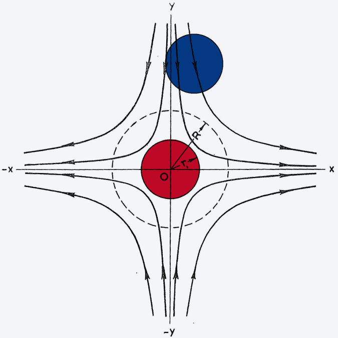
Saffman & Turner *JFM* (1956)

- ~~flow is locally hyperbolic~~
- ~~flow is persistent~~
- fluid element can pass through collision sphere repeatedly

$\Rightarrow$ spurious "ghost" collisions

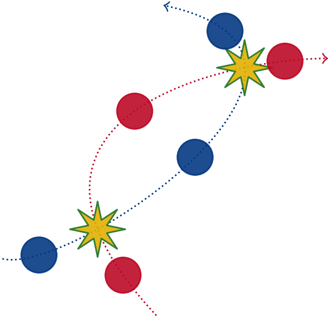
Brunk et al. *JFM* (1998)
Andersson et al. *EPL* (2007)
Gustavsson, Mehlig, & Wilkinson *NJP* (2008)

Ghost collisions and the Saffman–Turner theory



Saffman & Turner *JFM* (1956)

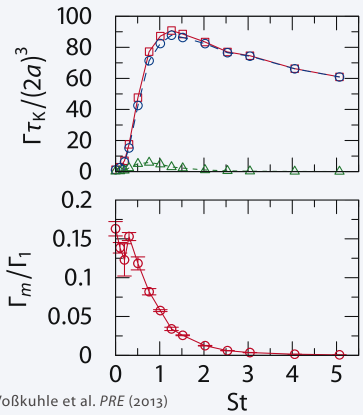
⇒



Particles kept in flow may collide again

Brunk et al. *JFM* (1998)
Andersson et al. *EPL* (2007)
Gustavsson, Mehlig, & Wilkinson *NJP* (2008)

Overestimation by Ghost collisions



Voßkuhle et al. *PRE* (2013)

Γ_{GCA} : ghost collision approximation

Γ_1 : only first collisions

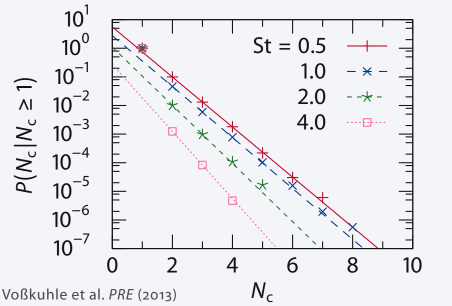
Γ_m : multiple collisions

$$\Gamma_{GCA} = \Gamma_1 + \Gamma_m$$

- Ghost collisions overestimate collision kernel by up to 20 %
- Relative estimation error falls with Stokes number

Consistent with previous results by Brunk et al. *JFM* (1998) and Wang et al. *Phys. Fluids* (1998)

Statistics of multiple collisions



Voßkuhle et al. *PRE* (2013)

- Multiple collisions statistics follow

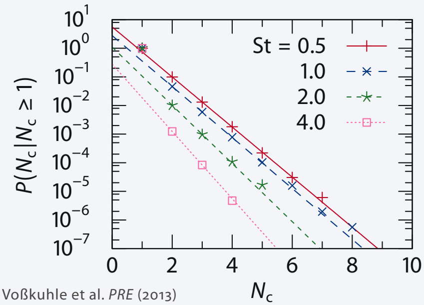
$$P(N_c|N_c \leq 1) = \beta(\mathrm{St})\alpha(\mathrm{St})^{N_c}$$

- Markovian interpretation: particle has probability $\alpha(\mathrm{St})$ to collide again

- Simple model

$$\Gamma_m = \Gamma_1 \sum_{N_c=2}^{\infty} \beta\alpha^{N_c} = \Gamma_1 \frac{\beta\alpha^2}{1-\alpha}$$

Statistics of multiple collisions



Voßkuhle et al. *PRE* (2013)

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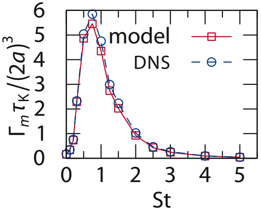
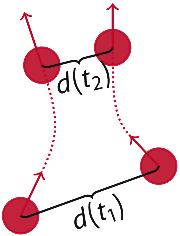
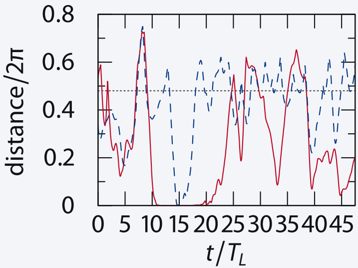
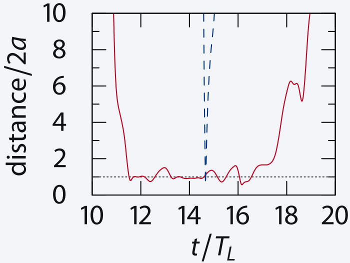


Distance between trajectories



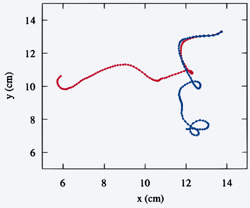




St = 1.0

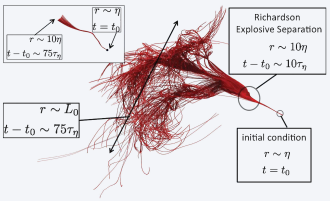
Pair separation

Jullien et al. *PRL* (1999)

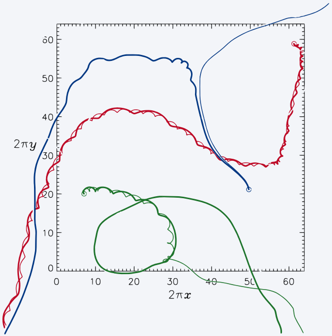


- results for tracers
- starting with similar initial conditions

Scatamacchia et al. *PRL* (2012)



Rast & Pinton *PRL* (2011)



Pair separation

- results for tracers
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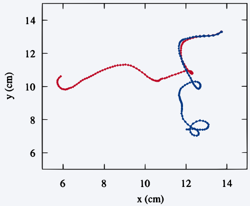
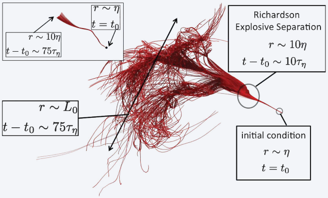
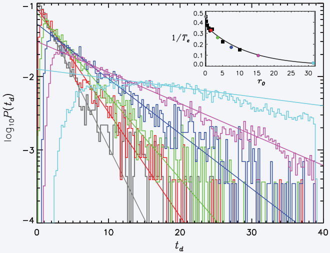
Jullien et al. *PRL* (1999)



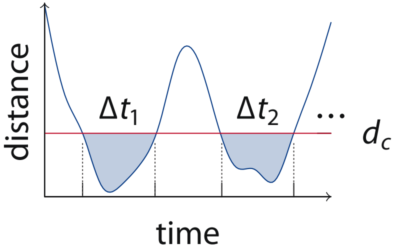
Scatamacchia et al. *PRL* (2012)



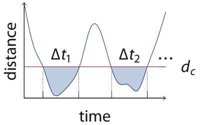
Rast & Pinton *PRL* (2011)



Contact time distribution

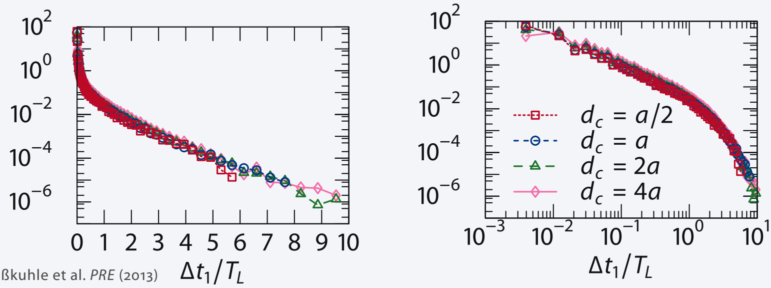


Contact time distribution



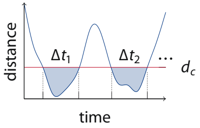
- exponential tail for long time
- power law for short time
- independent of distance d_c

$P(\Delta t_1)$ for St = 1.5



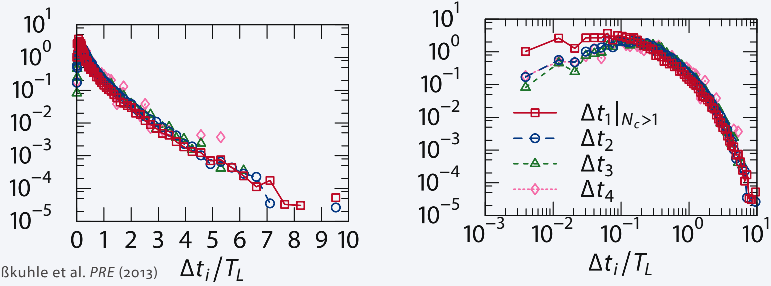
Voßkuhle et al. *PRE* (2013)

Contact time distribution



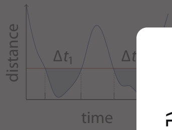
- exponential tail persists
- power law vanishes
- independent of collision count

$P(\Delta t_i)$, $i > 1$ for St = 1.5



Voßkuhle et al. *PRE* (2013)

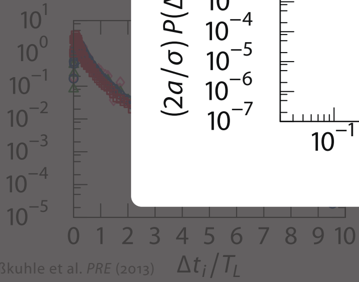
Contact time distribution

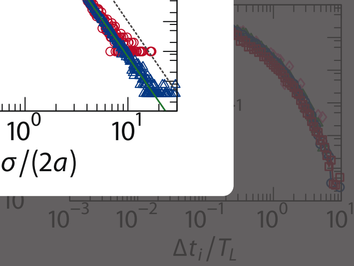


› exponential tail persists

...unt

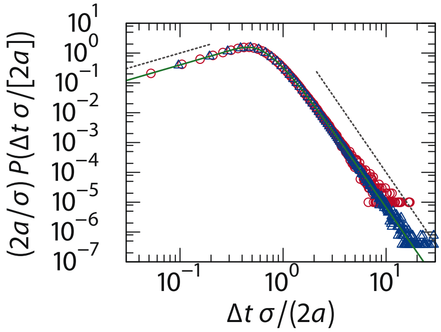
$P(\Delta t_i), i > 1$



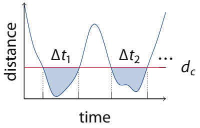


Voßkuhle et al. *PRE* (2013)

Kinetic theory

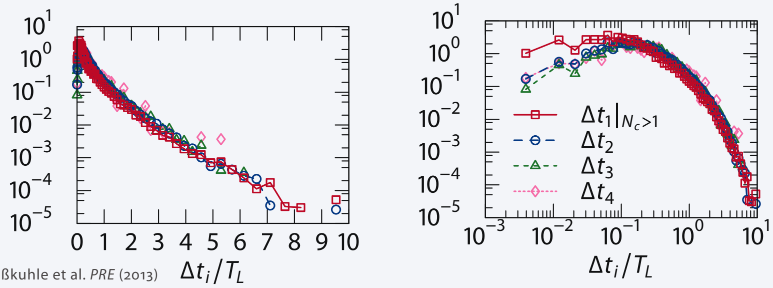


Contact time distribution



- exponential tail persists
- power law vanishes
- independent of collision count

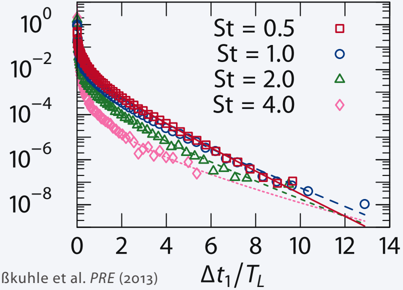
$P(\Delta t_i)$, $i > 1$ for St = 1.5



Voßkuhle et al. *PRE* (2013)

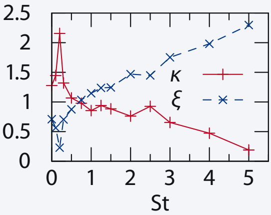
Contact time distribution

Different St



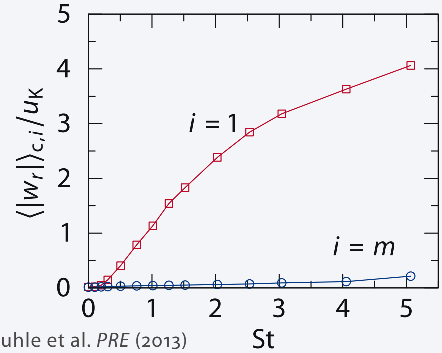
Voßkuhle et al. *PRE* (2013)

$$P(\Delta T_1) \sim e^{-\kappa \Delta t_1/T_L} \left(\frac{\Delta t_1}{T_L}\right)^{-\xi}$$



Multiple collisions and sling/caustics/RUM effect

Collision velocities



Voßkuhle et al. *PRE* (2013)

- multiple collisions have small relative velocities
- multiple collisions stem from continuous collisions
- sling/caustics/RUM effect does not lead to multiple collisions

Kinematic Simulations vs. Direct Numerical Simulations

Kinematic Simulations

$$\boldsymbol{u}(\boldsymbol{x}, t) = \sum_{n=1}^{N_k} \boldsymbol{A_n} \cos(\boldsymbol{k}_n \cdot \boldsymbol{x} + \omega_n t) + \boldsymbol{B}_n \sin(\boldsymbol{k}_n \cdot \boldsymbol{x} + \omega_n t)$$

$$\boldsymbol{A}_n \cdot \boldsymbol{k}_n = \boldsymbol{B}_n \cdot \boldsymbol{k}_n = 0$$

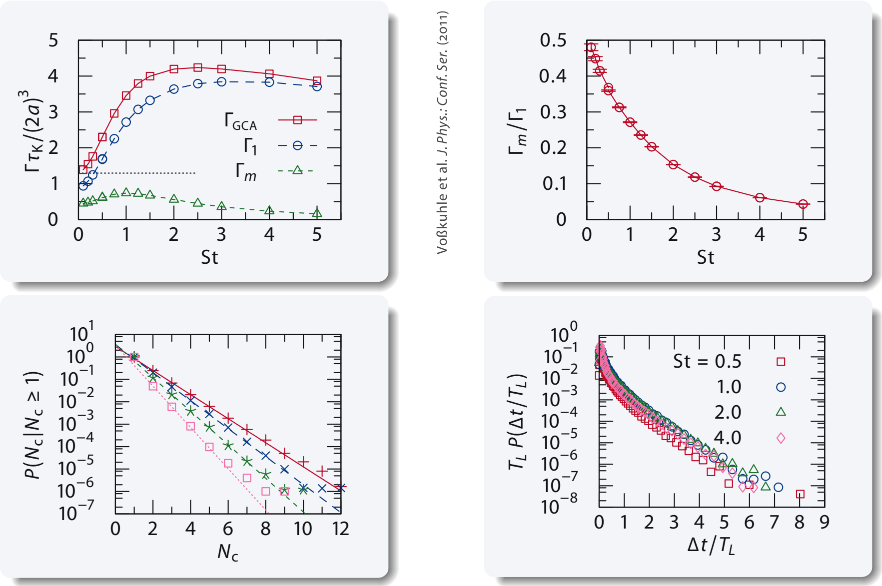
$$A_n^2 = B_n^2 = E(k_n)\Delta k_n$$

$$E(k_n) \sim k_n^{-5/3}$$

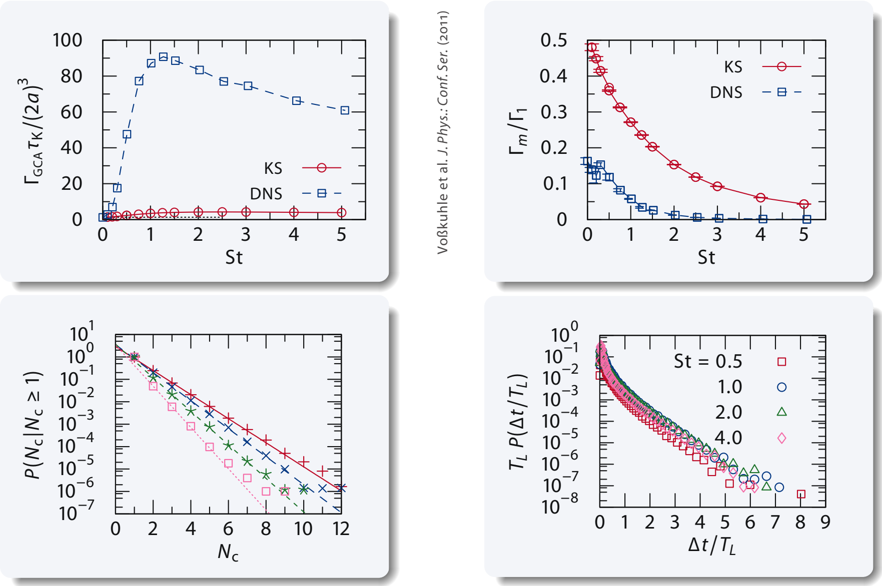
- efficient
- highly “turbulent” flows
- widely used
- “toy model”

Fung et al. *JFM* (1992)

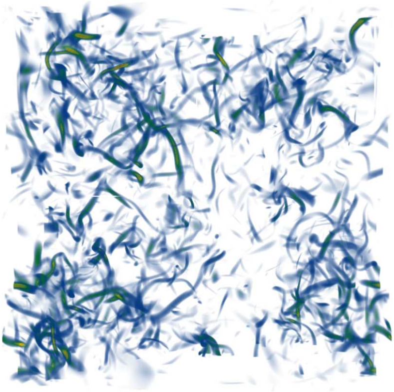
Qualitatively the same for KS…



…but quantitatively very different from DNS



Voßkuhle et al. *J. Phys.: Conf. Ser.* (2011)

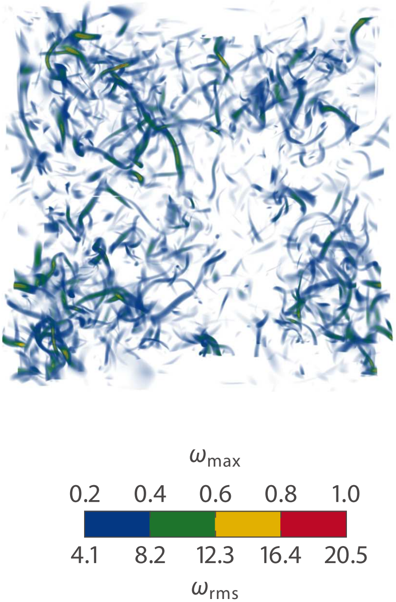


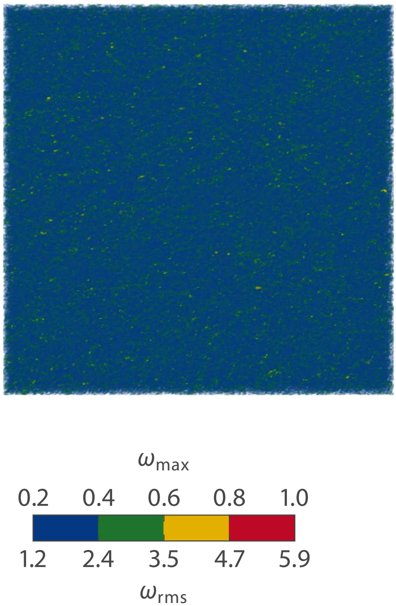
ω_{max}

0.2 0.4 0.6 0.8 1.0

4.1 8.2 12.3 16.4 20.5

ω_{rms}


ω_{max}
0.2 0.4 0.6 0.8 1.0
4.1 8.2 12.3 16.4 20.5
ω_{rms}


ω_{max}
0.2 0.4 0.6 0.8 1.0
1.2 2.4 3.5 4.7 5.9
ω_{rms}

A. Pumir

E. Lévêque

ED 52 - PHAST
Physique & Astrophysique de Lyon
Ecole doctorale

UNIVERSITÉ DE LYON

J. Bec M. Lance B. Mehlig M. Reeks M. Wilkinson

Laboratoire de Physique

PSMN
PÔLE SCIENTIFIQUE DE MODÉLISATION NUMÉRIQUE

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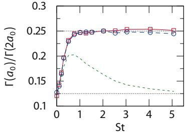
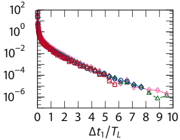
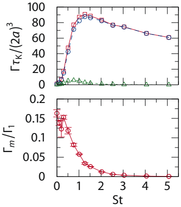
Conclusions and perspectives

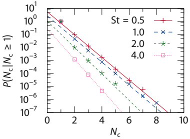
› St > 0: Sling/caustics/RUM effect dominates collision rates in turbulent flows



› inertial particles may stay close for long times







» leads to multiple collisions

» overestimation of the collision kernel

M Voßkuhle et al. (2013a). “Multiple collisions in turbulent flows.” In: *Phys. Rev. E* 88, p. 063008
M Voßkuhle et al. (2013b). “Prevalence of the sling effect for enhancing collision rates in turbulent suspensions.” In: *ArXiv e-prints* (July 2013). arXiv: 1307.6853 [physics.flu-dyn]
M Voßkuhle et al. (2011). “Estimating the Collision Rate of Inertial Particles in a Turbulent Flow: Limitations of the ‘Ghost Collision’ Approximation.” In: *J. Phys.: Conf. Ser.* 318, p. 052024

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Kinematic Simulations

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$$\boldsymbol{A}_n \cdot \boldsymbol{k}_n = \boldsymbol{B}_n \cdot \boldsymbol{k}_n = 0$$

$$A_n^2 = B_n^2 = E(k_n)\Delta k_n, \qquad E(k_n) \sim k_n^{-5/3}$$

$$k_1 = \frac{2\pi}{L}, \quad k_{N_k} = \frac{2\pi}{\eta}, \quad k_n = k_1 \left(\frac{L}{\eta}\right)^{(n-1)/(N_k-1)}$$

$$\omega_n = \lambda\sqrt{k_n^3 E(k_n)}, \quad \lambda : \text{“persistence parameter”}$$

- efficient
- highly “turbulent” flows
- widely used

Fung et al. *JFM* (1992)

› Go back…

- Bibliothèque municipale de Lyon. *Photo: Vélo'Vs in snow.*
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- http://commons.wikimedia.org. Some of the pictures are in the public domain and were taken from this website.
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